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THE INTEGRATION OF A PROBLEM-SOLVING FRAMEWORK FOR BRUNEI HIGH SCHOOL MATHEMATICS CURRICULUM IN INCREASING STUDENT'S AFFECTIVE COMPETENCY

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Abstract

A mathematics framework was developed to integrate problem-solving that incorporated simulation of real-life problems in the classrooms. The framework coined as the *RECCE-MODEL* emphasised understanding and thinking with a view on mathematics embedded in real-life. The *RECCE* which stands for Realistic, Educational, Contextual, Cognitive, and Evaluation encompass the underlying principles of teaching problem solving and guide teachers in planning, designing, developing, and facilitating real-life activity tasks in developing students' problem-solving competencies in mathematics lessons. It also explores students' cognitive competency in their application of abstract mathematical knowledge into real-life problems based on students' developmental status of their thinking and reasoning skills correlating to Meanings, Organise, Develop, Execute and Link (*MODEL*). This study investigated the affective development of the students through activity tasks developed by the sampled teachers using the principles within the framework. In total, 94 students from two high schools in Brunei Darussalam responded to a students' questionnaire constructed to address the *MODEL* aspect of the framework. In particular, the analyses involved the students' affective competencies that corresponded to a 19-item instrument within the questionnaire. The findings showed that Brunei high school students have stimulated beliefs and positive attitudes towards non-routine problem-solving in the learning of mathematics. Meanwhile, meaningful activities developed by the teachers encouraged the development of cognitive-metacognitive and affective competencies of the students. The *RECCE-MODEL* framework paved the way towards understanding the relationships between effective pedagogical approaches and students' learning, and between attitudes and cognitive abilities, and also for teachers to make better-informed decisions in the delivery of the curriculum.

Keywords: Mathematics Framework, Problem-Solving, Curriculum, Affective Competencies

Abstrak

Sebuah kerangka kerja matematika telah dikembangkan untuk mengintegrasikan pemecahan masalah yang menggabungkan simulasi masalah kehidupan nyata ke dalam pengajaran dan pembelajaran di kelas. Kerangka kerja yang diwujudkan sebagai *RECCE-MODEL* menekankan pemahaman dan pemikiran dengan pandangan tentang matematika yang tertanam dalam kehidupan nyata. *RECCE* yang bermakna Realistik, Pendidikan, Kontekstual, Kognitif, dan Penilaian merangkumi prinsip-prinsip asas mengajar pemecahan masalah dan membimbing guru dalam merancang, merekabentuk, membangun, dan memfasilitasi pembuatan tugas aktivitas dari kehidupan nyata dalam membangunkan kompetensi pemecahan masalah siswa dalam pelajaran matematika. Kerangka kerja tersebut juga mengeksplorasi kecekapan kognitif siswa dalam penerapan pengetahuan matematika yang abstrak ke dalam masalah kehidupan nyata berdasarkan status perkembanganpemikiran dan penalaran siswa yang berkaitan dengan Pengertian, Mengorganisasi, Membangun, Melaksana dan Menghubungkan (*MODEL*). Kajian ini menginvestigasi perkembangan afektif siswa melalui tugas-tugas aktivitas yang dikembangkan oleh guru-guru menggunakan prinsip-prinsip dalam kerangka kerja ini. Secara keseluruhan, 94 siswa dari dua sekolah menengah di Brunei Darussalam menanggapi kuesioner siswa yang dibangun untuk membahas aspek MODEL dari kerangka kerja. Secara khusus, analisis melibatkan kompetensi afektif siswa yang sesuai dengan instrumen 19 item dalam kuesioner. Penelitian menemukan bahawa siswa sekolah menengah di Brunei telah menstimulasi keyakinan dan sikap positif terhadap pemecahan masalah yang tidak rutin dalam pembelajaran matematika. Sementara itu, aktivitas-aktivitas yang bermakna yang dikembangkan oleh guru-guru dapat mendorong pengembangan kecekapan kognitif-metakognitif dan afektif siswa. Kerangka kerja *RECCE-MODEL* membuka jalan ke arah pemahaman hubungan antara pendekatan pedagogi yang efektif dan pembelajaran siswa, dan antara sikap dan kemampuan kognitif, dan juga untuk guru membuat keputusan yang lebih bijak dalam penyampaian kurikulum.

Kata kunci: Kerangka Kerja Matematika, Pemecahan Masalah, Kurikulum, Kompetensi Afektif

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Mathematical modelling is one of the applied mathematical tools that support real-life problem solving in mathematics education that has emerged from several perspectives. Blomhøj (2008) identified five main perspectives of research on the teaching and learning of mathematical modelling: 1) The realistic perspective – authenticity of real life modelling in designing problems where students learning is supported by relevant technology, and assess the model and its results against the reality; 2) The epistemological perspective –the development of more general theories and practices in the teaching and learning of mathematics; 3) The contextual perspective – to include research on problem solving and deepening the philosophical role of word problems in its connection to learning theories; 4) The cognitive perspective – students' modelling processes are analysed with the purpose of understanding the cognitive functions and cognitive barriers of the individual going through the modelling process; and 5) The educational perspective–integrating mathematical modelling in the teaching of mathematics, and discuss problems related to assessing students' learning processes using mathematical modelling activities from different types of mathematics curricula.

Barbosa (2012) adopted mainly the education perspective in Brazil where the focus of learning mathematical concepts and the development of 'modelling competencies' are viewed as a way to teach mathematical concepts, in relation to the idea that mathematics education must take part in efforts to educate students be critical, engaged citizens. In the 21st century, it is not sufficient for students to be only competent in applying mathematical knowledge in the context of the framework of the curriculum, which describe the cognitive and educational perspectives. Instead wider perspectives that include embedding real world contexts into the curriculum are needed to support students' cognitive development in engaging new ideas, supporting earlier understandings, and mathematical reasoning from abstraction to solutions. Consequently, it would be appropriate to adapt all five perspectives proposed by Blomhøj (2008) in developing the mathematics framework for Brunei mathematics education. Our teachers need not only teach the curriculum, but continuous support and guidance from relevant stakeholders in educating the future generation is crucial, especially the kind of support and guidance that may elicit confidence and relevance in raising the quality of teaching and learning. Thus, one of the way forward for our mathematics education will be to have our own relevant framework, which guides teachers in preparing their lessons that is realistic, educational, contextually relevant, cognitively challenging for their students.

The Mathematics Framework: RECCE-MODEL

Anthony and Walshaw (2009) identified ten principles of effective mathematics pedagogy, namely an ethic of care, arranging for learning, building on students' thinking, worthwhile mathematical tasks, making connections, assessment for learning, mathematical communication, mathematical language, tools and representations and teacher knowledge, that were found to develop mathematical capability and disposition within an effective learning community. They believed that holistic development of productive students depends highly on effective mathematics pedagogy, which acknowledges the mathematical potentials in *all* students in optimising a range of desirable academic outcomes, and also enhancing a range of social outcomes in classroom. Thus, the ten principles encompass the complex dynamic of a classroom environment within the western education system, where the nature of classroom mathematics teaching focus mainly on students' learning in a safe and supportive environment. This corresponds highly to Brunei's current education system model entitled the National Education System for the 21st Century or *Sistem Pendidikan Negara Abad Ke*-21 or termed as SPN21 (Ministry of Education, 2013). Accordingly, the primary goal of the SPN21 curriculum is based on the principle that each learner is the centre of all teaching and learning through the process of knowledge and understanding, essential skills, and attitudes and values in a well-balanced education system.

In conducting a lesson on problem solving, Lester, Garofalo and Kroll (1989) also proposed that teachers focus on creating a classroom culture of mathematical inquiry through connection and relevant discourse. A design by Lester, Garofalo and Kroll (1989) was also explored to study the effect of instruction on students' cognitive self-regulation of the problem solving processes. This also helps to build the foundation of the current framework. In addition to the ten principles of effective teaching by (Anthony & Walshaw, 2009), the five perspectives proposed by Blomhøj (2008), and the additional five fundamental elements of education by Novak (2013a, 2013b), which are the learner, the teacher, the curriculum, the context, and evaluation, had been incorporated in developing the present mathematics framework. Novak (2013a, 2013b) recognised that in enhancing any successful educational event, each of these five elements must be optimised. Underpinning these principles and perspectives; Pólya's Model (1945), Garofalo and Lester (1985) cognitive and metacognitive framework, Carlson and Bloom (2005) *Mathematical Problem Solving* (*MPS*) framework and modelling cycle by Blum and Leiβ (2007), an emerging mathematics framework representing *Realistic*, *Educational*, *Contextual*, *Cognitive, and Evaluation* - *RECCE* and *Meanings, Organise*, *Develop, Execute*, *Link* - *MODEL* (see Figure 1) was developed for this present study applicable to the mathematics curriculum of Brunei.

The *RECCE-MODEL* is a framework developed to encompass the underlying principles of teaching problem solving by incorporating simulation of real-life problems in classrooms, which emphasized contextually relevance, understanding and expressing thinking with a view on mathematics embedded in real-life. Furthermore, the framework sets direction in learning and assessment of mathematical knowledge and skills in developing students' cognitive, metacognitive and affective competencies. *RECCE* aims to guide teachers in planning and designing their mathematics lessons, developing non-routine activity tasks and evaluate the implementation process of the lesson plans to subsequently make improvement. It is important that the assessment of the learning process in providing information about the progress of students in achieving learning goals are conducted through learning activities between the teacher and the student (Kenedi, *et al*. 2019; Shahrill & Prahmana, 2018; Khoo, *et al*. 2016). The *RECCE-MODEL* framework also echoed similar importance between teaching problem solving and developing competencies through the use of real-life activities and eventually achieving the learning goals. Therefore, this aspect of the framework is focusing on the structuring and development of meaningful lessons to maximize learning in the classroom. Two theoretical perspectives were drawn in developing the conceptual design of the *RECCE-MODEL* framework. Both constructivism and Ausubel's (2000) assimilation of cognitive learning provided the theoretical perspectives in guiding this present study.

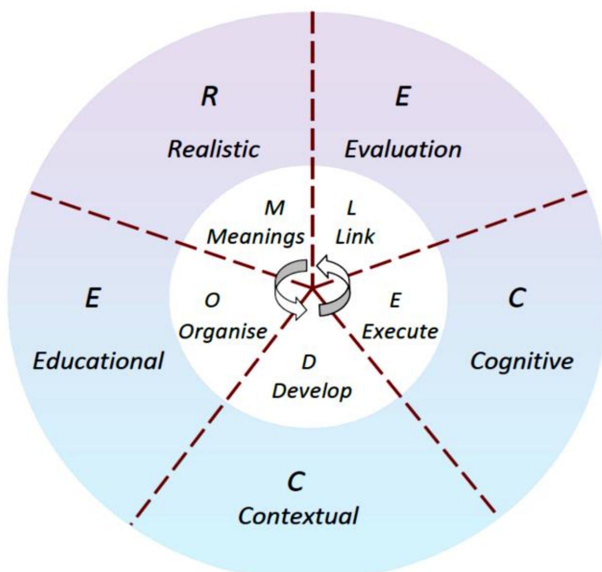

Figure 1. The emerging RECCE-MODEL mathematical problem-solving framework

From Figure 1, the *R*ealistic principle of the framework plays an important role in developing students' cognitive and metacognitive competencies. Teachers are to design lessons focusing on non-routine problems that will develop students' mathematical problem solving skills and thinking skills. This may further improve students' conceptual understanding, application of abstract mathematics and encourage students to be self-aware and regulate own thinking. The *E*ducational principle covers the mathematics curriculum set out by the Ministry of Education. The teachers are to create learning experiences to develop students' understanding of concepts, ideas and applications as an integrated whole process of learning mathematics. Students are encouraged to participate actively in exploring and learning mathematics using worked examples, activities, tasks, and technological aids. The *C*ontextual principle refers to the ability to connect within mathematical concepts and ideas; and also interdisciplinary. This is to help students engage with real problems and make sense of what they are learning, through connecting ideas and regulating thinking precisely, logically and concisely. The *C*ognitive principle is to develop students' thinking skills through seeking solutions, exploring patterns, and formulating conjectures. Students are encouraged to communicate and share their ideas and methods of workings to others as a way of developing their communicating skills. The *E*valuation principle refers to teachers reflecting their teaching approaches and lessons conducted to effectively improve students' competencies in learning and applying mathematics. In addition, the earlier four principles (*R*ealistic, *E*ducational, *C*ontextual and *C*ognitive) must be reviewed to subsequently make improvement in designing lessons that contribute to the success of teaching and learning mathematics.

The *RECCE-MODEL* framework proposes that teachers create a mathematics classroom based on the five guiding principles of *RECCE,* to engage students in mathematical thinking and problem solving through constructivist approach. This is the approach where knowledge is constructed by learners in new experiences from previous learning and propositions of the learning environment, which leads to deeper understanding and flexibility in their mathematical thinking. The key elements of the teacher's role involved planning an overall course of lesson plans; selecting appropriate resources and mathematical problems following the three fundamental requirements for meaningful learning by Novak (2013a, 2013b); monitoring process and progress; and evaluating results.

Therefore, the *RECCE-MODEL* framework aims to create a strong link between teachers' approaches to specifying the mathematical problem solving processes from mathematical content of the curriculum to the mathematical reasoning required in problem solving. Teachers are also expected to foster classroom climate that includes non-routine tasks which, enhances students' beliefs and affects in further contributing to their metacognitive competency towards successful problem solving.

Meanwhile, the *MODEL* framework is used to examine and evaluate students' cognitive-metacognitive competencies in completing a mathematical task. While, students also used *MODEL* in assessing their Level of competencies in completing a task through creating meaning from the real-life problem posed (Level 1); identifying the dependent and independent variables in the problem posed (Level 2); deciding which variables and appropriate mathematical formulae are feasible and possible to use in solving the problem (Level 3); obtaining mathematical solution(s) and contextualise the solution(s) in order to justify for interpretations (Level 4); and finally linking to validate the solution(s) to the problem and reflecting on any error(s) encountered (Level 5). Furthermore, the *MODEL* framework explores students' cognitive competency in six levels, in their application of abstract mathematical knowledge into real-life problems based on students' developmental status of their thinking and reasoning skills correlating to *Meanings*, *Organise*, *Develop*, *Execute* and *Link* (*MODEL*) (shown in Table 1).

Table 1. MODEL cognitive-metacognitive framework categorised in 6 levels in performing a mathematical task

Competency Level & Category	Cognitive-Metacognitive Competency Learners	Key Feature
L0 – No attempt	Did not attempt the problem.	Neither working nor solution is correctly shown.
L1 – *M*eanings	Recall existing propositions; Attempt to make connections; Attempt to make assumptions; Analyse and make meanings of the problem – Understand the problem.	Knowing *about* the problem.
L2 – *O*rganise	Exploring the propositions; Identify strategies; Identify dependent and independent variables; Reflect back to L1.	Knowing *how* to apply.
L3 – *D*evelop	Formulating strategies and variables; Understanding the mathematical concepts needed to solve the problem; Develop a plan; Consolidate L1 and L2.	Knowing *which* to apply.
L4 – *E*xecute	Implement strategies and variables; Monitor progress of the implemented plan; Consolidate L1, L2 and L3 to obtain solution(s) to the mathematical problem.	Knowing *what* and *when* to apply.
L5 – *L*ink	Reflect back solution(s) to the problem; Interpret solution(s) to the problem; Monitor consistency of solution(s); Monitor consistency of plan; Start again if necessary.	Knowing *why* it is applied.

In L1 – *Meanings* (M), students must present some fragments of their abstract knowledge into diagrammatic representation of the problem using concept map, mind map, flowchart, diagrams of all sorts and also any relevant figures. At this Level, students will demonstrate memory recall and reinforced prior knowledge or learning into the real-life problem posed. In L2 – *Organise* (O), students must identify the dependent and independent variables in the real-life problems posed. They will explore and generate ideas, parameters and break down the problem into simpler task by asking questions and linking ideas. In L3 –

Develop (D), students make relevant assumptions based on their ideas and decide which variables are feasible and possible to solve this problem. Students will learn creative decision-making at this Level by choosing the appropriate mathematical formulae to use in solving the problem. In L4 – *Execute* (E), students will obtain mathematical solution(s) at this Level, and will need to contextualize the solution(s) in order to justify for interpretations at the final Level. The learning outcome at this Level is that students will demonstrate their metacognitive competency in reflecting back into the problem. And the fifth Level, L5 – *Link* (L), the metacognitive Level, and students must be able to link and validate their solution(s) to the problem and finally reflecting on any error(s) encountered.

The *MODEL* framework proposes that students to self-scaffolding by following the five levels of problem solving in helping them to become self-aware and self-regulate in their thinking, thus supporting their use of knowledge to help solve a problem. Therefore, with the development of the *RECCE-MODEL* framework, this study aims to investigate the affective development of the students through activity tasks (Chong, *et al*. 2018) developed by the sampled teachers using the principles within the framework. A pilot study was conducted in identifying the affective competencies of Brunei pre-university students (or high school equivalent of Year 12 in the United Kingdom or the 11th Grade in the United States), prior to the development of the *RECCE-MODEL* framework. The pilot study concluded that the affective competencies of Brunei students are stimulated and can be further developed through structured activities in a learning environment (Chong & Shahrill, 2015). Thus, the development of this framework will provide the structure in designing realistic, educational, contextual and cognitive challenging tasks to develop students' affective competencies.

METHOD

A mixed (qualitative and quantitative) research methodology was employed in this study, to engage teachers and students in working with *RECCE-MODEL* in integrating perspectives on problem solving of real-world examples through activity tasks (Chong, *et al*. 2018). The quantitative data were collected using a students' questionnaire, and the qualitative data gathered from semi-structured interviews involving all the participants using open-ended questions and were conducted in groups of four to six students, following the recommendation from Creswell (2013) in relation to focus group interviews. The questionnaire was designed in three sections: the first section consists of questions regarding students' demographic and academic characteristics; the second section consider students' perceptions of the five aspects of the *MODEL* framework; and the last section consider students' affective domain of learning mathematics (beliefs and attitudes).

The students' questionnaire was developed addressing the *MODEL* aspect of the framework and how it interconnects between students' cognitive and metacognitive competencies as they go through the process of problem solving. All the items developed also provided opportunities to critically reflect on individual's attitudes and beliefs of learning mathematics. The development of the questionnaire followed the requirement and criteria set out by Cohen, *et al*. (2011) to obtain as much personal information and academic background of the students as possible and also to assess students' affective competency in learning mathematics. The questions that are designed to capture students' affective competency are in rating scales following Likert scale ranging from never = 1 to always = 5. The design of the questionnaire was concise such that five items that describe the experience of doing and learning mathematics within the context represented each category of the MODEL framework. The questionnaire only required students to read the questions, read the possible responses and mark their responses accordingly. At the start of administering the questionnaire, for ethical considerations, students were informed and assured of the confidentiality, anonymity and non-traceability as all information and data were aggregated into categories. Piloting of the questionnaire was conducted in one of the pre-university institutions prior to implementing the main study.

Meanwhile, the use of activity tasks in this study was to enhance students' cognitive, metacognitive and affective capabilities through communication, self-regulation, and facilitating discovery in enhancing understanding of the problem, and thus supporting students' cognitive, metacognitive and affective development towards non-routine problem solving being part of their learning experiences in mathematics. The subsequent results of the pilot study was also reported in Chong and Shahrill (2016), and the findings showed that Brunei high school students have stimulated beliefs in learning of mathematics and positive attitudes towards non-routine problem solving being part of learning in mathematics.

In reporting the findings in this paper, the students' affective competencies were explored from their responses to a set of 19 questionnaire items that described their beliefs and attitudes towards mathematics and problem solving in general. The 19 items appeared at the last section of the students' questionnaire. In total, the sample size comprised of 94 students from which 42 students were from the first participating high school and the remaining 52 students were from the second high school. There were 33 male students (35.1%) and a total of 61 female students (64.9%). The participating students' ages ranged from 15 to 20 years old.

RESULTS AND DISCUSSION

The reliability score of the 19-items instrument was in the acceptable range of Cronbach's alpha value of 0.76. The results were confirmatory with all 19 items as they fit all the six dimensions in Table 2 below. The questionnaire of this present study was administered after the intervention has been completed. Therefore, the participating students' views of learning mathematics and problem solving in this study was reflective of their attitudes and beliefs after the intervention has been carried out. This was to measure the extent of how *RECCE-MODEL* helps to develop students' affective competency in relation to their cognitive and metacognitive development in solving non-routine problems.

McLeod (1989) viewed emotion as one of the critical factor influencing the process of solving non-routine mathematical problem. The emotion described by McLeod was the feeling of frustration with each unsuccessful attempt; the feeling of anger when a solution cannot be reached; and the feeling of satisfaction and joy when solution is obtained. Therefore, this domain of feelings described by McLeod plays a critical role in influencing the cognitive processes of solving problem, in particular non-routine problems. This is because the extent of the willingness of an individual to solve a problem is greatly dependent on the individual understanding of the problem posed, the kinds of decision-making made during the process and also the working conditions. Schoenfeld (1983) also presented similar views, where he discussed that students manage their cognitive resources through students' belief systems which, included attitudes towards mathematics and confidence about mathematics. Consequently, McLeod (1992) has re-conceptualized beliefs and attitudes towards mathematics as the affective domain in mathematics education and instruction. He categorized beliefs into *beliefs about mathematics* (importance, difficulty, and based on rules), *beliefs about self* (self-concept, confidence and metacognition), *beliefs about mathematics teaching or mathematics classroom instruction*, and *beliefs about the social context* (home environment, parental and peer influences).

Earlier work by Ernest (1988) has distinguished three conceptions of beliefs about mathematics teaching and learning into *the instrumentalist view, Platonist view and the problem-solving view*. The significance of these views is that a learner with *instrumentalist view* will view mathematics as collection of facts, skills and rules with no connection, *Platonist* will view mathematics as a static body of knowledge, and *problem-solving* learner will view mathematics as dynamic with content continually growing (Allen, 2010; Shahrill, *et al*. 2018). In her study, Allen discussed that teachers need to shift their views to one of the problem-solving view in order to be effective teachers of mathematics. Similarly, in the context for a student to be effective learner, one must view mathematics as a process of enquiry and exploration, not just mastery of facts and procedures.

Table 2. The six dimensions of the students' affective competency in learning mathematics and problem solving

Items	Value of Factor matrix	Dimensions (No. of Items Related to the six Dimensions)
1. I seek help from a mathematics tutor.	.745	Attitudes towards social context (3)
2. I seek help from peers (discussion to seek mathematical solutions).	.836	
3. I work in a group to solve mathematics problems.	.655	
4. I think mathematics is useful in everyday life.	.755	Beliefs about mathematics (4)
5. I think that mathematics is used in everyday life.	.749	
6. I use mathematics in everyday life.	.767	

7. I think mathematics will help in my future career path.	.386	
8. I am curious about the mathematical solutions obtained.	.867	Attitudes towards learning mathematics (2)
9. After completing a mathematics question, I try to interpret the solution(s).	.674	
10. I look forward to a mathematics lesson.	.758	
11. I think mathematics is fun to learn.	.839	Positive beliefs (4)
12. I am very keen to learn new ideas and theories in mathematics.	.661	
13. I usually do well in mathematics.	.519	
14. I work individually to solve mathematics questions.	.759	
15. I finished all assigned mathematics assignments.	.588	Self-beliefs (3)
16. I learn mathematics through understanding and problem-solving strategies.	.709	
17. I learn mathematics through memorising of formulae and procedures.	.688	
18. I think mathematics is all about solving equations (numerical computation).	.870	Instrumentalist beliefs (3)
19. I think mathematics solution is just a numerical number.	.709	

Presented in Table 3 are the descriptive statistics of the six dimensions of the students' affective competency in learning mathematics and problem solving. Entries from Table 3 were evident that students have strong beliefs about mathematics and also positive beliefs. These two dimensions recorded the highest mean values in comparison to its total maximum score.

Table 3. Numerical variables between the six dimensions of the students' affective competency

Six dimensions of students' perceptions	Total Minimum score	Total Maximum score	Mean (SD)
1. Attitudes towards social context	3	15	9.9 (1.96)
2. Beliefs about mathematics	4	20	16.4 (2.90)
3. Attitudes towards learning mathematics	2	10	6.5 (1.58)
4. Positive beliefs	4	20	14.7 (2.83)
5. Self-beliefs	3	15	11.3 (2.12)
6. Instrumentalist beliefs	3	15	10.6 (2.33)

These findings were further supported by students' comments from the interviews when asked these questions: why study mathematics and what use of mathematics is important for you to learn? The following are excerpts from the interviews that were relevant to support the findings:

T1 *For me, it is my best subject. I like it and it also gives **a lot of help in my other subjects**. Physics, there's all these Maths, also in Computer Science, there's all these calculations where we're converting numbers in a system to another system. It's very (cradles his head in his hands). Maths is definitely helping in all my other subjects and also one of my goals.*
H1 *Because one, it's **easy** and-second, it's **most job requirements**.*
X1 *I take Maths because it's **essential for life**.*
Z1 *I take Maths because I like Maths and also we get **more job opportunities.***
L1 *Because you need Maths to get a… Because many **university subjects require Maths.***
A1 *It's **fun**.*
A1 *Hehe. Because we… we need- we **need Maths** in every- **in our everyday life.***
I1 *Because it can **help in my career** in the future.*
N1 *It would be **useful for my Economics** because I plan to take Economics degree*
C1 *I take Maths because it is **important.** Because it is **related to Physics**.*
F1 *Hmmm. I find that it is **interesting** and sometimes I can **release my stress** by just doing the past year questions.*
B1 ***I love maths**, and I think I'm good in maths and that's why I'm doing Maths.*
F1 *When I ask my friend, they say that maths is really important when you want to get a job. Nowadays, I think it is the **most important subject**.*
D1 *Because **I like mathematics** and doing calculations*
G1 *Because my father said... Like, **maths is important for all**. Like, any course you want to take. Maths is important.*
V1 *Because it might be **helpful in the future**.*
O1 *I have the **interest** to study Maths in A-Level. It's actually because of **my career**. I have two career basically either become engineering or the doctor. So to be engineering, engineer, so I need to take Maths.*
K1 *Basically we **use mathematics everyday** either we do realize or not so if we **don't have knowledge in Maths we will be lost** in such a ways…*
P1 *Uhh because **I love Maths**.*
Q1 *Uhh the reason why I take Maths is because I want to **pursue law**, for my University course and I did a lot of research about requirements what I should take for my A' Level subjects to pursue Law and most of it says that it is better for me to take Maths, History and English Lit and it's also a bonus point that I enjoy all of my subjects, I enjoy Maths so pretty much why I'm taking it.*
U1 *I think Maths is basically **useful in our everyday life** and I'm also interested in becoming a chemical engineer, so that needs Maths.*
P1 *Because **Maths is important for life**.*

Further analysis of the interview excerpts showed that the participating high school students have very strong perception of the purpose and importance of learning mathematics. They believed that mathematics will be able to support their future career paths and is essential for life. During the intervention, the mathematics pedagogical approaches developed by the sampled teachers using *RECCE-MODEL* framework have shaped the students' beliefs and their behaviour in learning problem solving. In particular, the teachers' actions in scaffolding students' learning during the interactions using the activity tasks (Chong, *et al*. 2018), the technology, the resources and their peers, were crucial to the success of solving the tasks. This seemingly simple findings have important implications on how students learn and apply the metacognitive processes and strategies during the activity tasks. For example, a task on designing a school car park was viewed as the most challenging task for majority of the students, but it enriched their metacognitive experience as the students continually check the appropriateness of their solutions and justifying the final solution. The task was designed to give students the opportunities to reflect on their strategies following their engagement in the problem-solving task with their group members. Furthermore, they had to test, redesign if necessary and review their solutions repeatedly during the problem solving process guided by the *MODEL* framework. Consequently, all groups persevered and managed to complete this task through good discussion and strategic collaboration. This was attributed by the teachers' influences on changing the culture of the classroom by bringing the *realistic* experiences of learning mathematics through non-routine problem solving in the classroom.

CONCLUSION

The sampled teachers in this study provided meaningful tasks that encouraged the development of cognitive-metacognitive and affective competencies of the students. The progress of the *RECCE-MODEL* framework has paved the way towards understanding the relationships between effective pedagogical approaches and students' learning, and between attitudes and cognitive abilities, and also for teachers to make better informed decisions in the delivery of the curriculum. Goos, *et al*. (2017) identified mathematical knowledge base, heuristics, self-awareness, self-regulation, beliefs, affects and classroom environment are the factors that contribute to successful problem solving. And these factors are inter-connected to one another. Evidently, a teacher plays a critical role in shaping students' beliefs and attitudes towards a learning environment. Therefore, a simple change in teachers' classroom practice in this study appeared to influence and articulate students' beliefs and dispositions in deepening their mathematical engagement. Through synthesis of researches, Lesh and Zawojewski (2007) pointed out that developing a productive problem-solving persona involves complex, flexible, and manipulatable profile of affect. Therefore, co-developing affective and metacognitive competency can contribute to how cognition develops in learning mathematics. Sari and Mutmainah (2018) also highlighted similar significance of teacher's role in delivering the subject matter to motivate learning of mathematics for students through creative, open and joyful learning. It can be suggested that with high cognitive demand tasks, students may be more engaged and become active in the exploration stage, and may be able to use strategies that were meaningfully connected to concepts. To conclude, this present study marked the beginning of integrating a mathematics framework called the *RECCE-MODEL* into the Brunei school curriculum in developing students' affective competencies in the learning of mathematics.

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